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SCALABLE QUANTUM NETWORKS BASED ON FEW-QUBIT REGISTERS

LIANG JIANG

Center for the Physics of Information and Institute for Quantum Information,
California Institute of Technology, Pasadena, CA 91125, USA
ljiang@caltech.edu

JACOB M. TAYLOR

National Institute of Standards and Technology,
Gaithersburg, Maryland 20899, USA
jacob.taylor@nist.gov

ANDERS S. SØRENSEN

Quantop and The Niels Bohr Institute, University of Copenhagen,
DK-2100 Copenhagen Ø, Denmark
anders.sorensen@nbi.dk

MIKHAIL D. LUKIN

Department of Physics, Harvard University,
Cambridge, Massachusetts 02138, USA
lukin@fas.harvard.edu

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We describe and analyze a hybrid approach to scalable quantum computation based on an optically connected network of few-qubit quantum registers. We show that probabilistically connected five-qubit quantum registers suffice for deterministic, fault-tolerant quantum computation even when state preparation, measurement, and entanglement generation all have substantial errors. We discuss requirements for achieving fault-tolerant operation for two specific implementations of our approach.

Keywords: QND measurement; entanglement purification; finite-state Markov chain.

1. Introduction

The key challenge in experimental quantum information science is to identify isolated quantum mechanical systems with good coherence properties that can be manipulated and coupled together in a scalable fashion. Substantial progress has been made towards the physical implementation of few-qubit quantum registers using systems of coupled trapped ions,[1–4] superconducting islands,[5,6] solid-state qubits based on electronic spins in semiconductors,[7] and color centers in diamond.[8–12] While the precise manipulation of large, multi-qubit systems still remains an outstanding challenge, approaches for connecting such few qubit registers into large scale circuits are currently being explored both theoretically[13–18] and experimentally.[19,20] Of specific importance are approaches which can yield fault-tolerant operations with minimal resources and realistic (high) error rates.

In Ref. 14, a novel technique to scalable quantum computation was suggested, where high fidelity local operations can be used to correct low fidelity non-local operations, using techniques that are currently being explored for quantum communication.[21–23] In this paper, we present an architecture, which requires only five (or fewer)-qubit registers with local deterministic coupling. We report the following major results with detailed derivation provided in Ref. 24. The small registers are connected by optical photons, which enables non-local coupling gates and reduces the requirement for fault tolerant quantum computation.[25] Besides providing additional improvements over the earlier protocol[14] (suppressed measurement errors, more efficient entanglement purification, and higher final entanglement fidelity), we analyze two physical systems where our approach is very effective. We consider an architecture where pairwise non-local entanglement can be created in parallel, as indicated in Fig. 1. This is achieved via simultaneous optical excitation of the selected register pairs followed by photon-detection in specific channel. We use a Markov chain analysis to estimate the overhead in time and operational errors, and discuss the feasibility of large scale, fault-tolerant quantum computation using this approach.

The present work is motivated by experimental advances in two specific physical systems. Recent experiments have demonstrated quantum registers composed of few trapped ions, which can support high-fidelity local operations.[2–4] The ion qubits can couple to light efficiently[29] and were recognized early for their potential in an

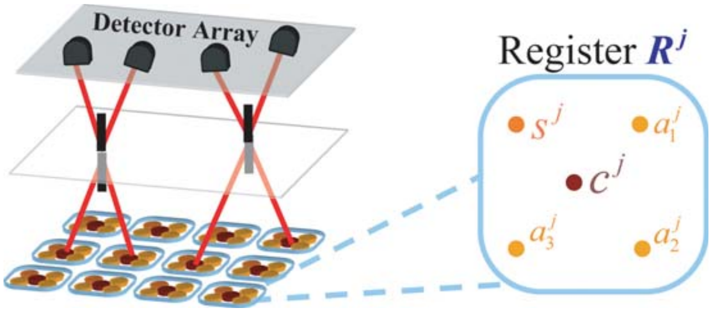

Fig. 1. Illustration of distributed quantum computer based on many quantum registers. Each register has five physical qubits, including one communication qubit (c), one storage qubit (s), and three auxiliary qubits ($a_{1,2,3}$). Local operations for qubits from the same register have high fidelity. Entanglement between remote registers can be generated probabilistically.[23,26,27] Optical microelectromechanical systems (MEMS) devices[28] can efficiently route photons and couple arbitrary pair of registers. Detector array can *simultaneously* generate entanglement for many pairs of registers.

optically coupled component.[14,18] Probabilistic entanglement of remote ion qubits mediated by photons has also been demonstrated.[30,31] At the same time, few-qubit quantum registers have been recently implemented in high-purity diamond samples.[9–12] Here, quantum bits are encoded in individual nuclear spins, which are extraordinarily good quantum memories[11] and can also be manipulated with high precision using techniques from NMR.[32–34] The electronic spin associated with a nitrogen-vacancy (NV) color center enables addressing and polarization of nuclei, and entanglement generation between remote registers. While for systems of trapped ions there exist several approaches for coupling remote few-qubit registers (such as those based on moving the ions[35]), for NV centers in diamond it is difficult to conceive a direct construction of large scale multi-qubit systems without major advances in fabrication technology. For the latter scenario the hybrid approach developed here is required. Furthermore the use of light has the major advantage that it allows for connecting spatially separated qubits, which reduces the requirement for fault-tolerant quantum computation.[25]

2. Quantum Registers

We define a *quantum register* as a few-qubit device that contains one *communication* qubit, with a photonic interface; one *storage* qubit, with very good coherence times; and several *auxiliary* qubits, used for purification and error correction (described below). A critical requirement for a quantum register is high-fidelity unitary operations between the qubits within the register.

The simplest quantum register requires only two qubits: one for storage and the other for communication. Entanglement between two remote registers may be generated using probabilistic approaches from quantum communication (Ref. 23 and references therein). In general, such entanglement generation produces a Bell state of the communication qubits from different registers, conditioned on certain measurement outcomes. If state generation fails, it can be reattempted until success, with an exponentially decreasing chance of continued failure. When the communication qubits (c^1 and c^2) are prepared in the Bell state, we can immediately perform the remote C-NOT gate on the storage qubits (s^1 and s^2) using the gate-teleportation circuit between registers R^1 and R^2. This can be accomplished[18,36–39] via a sequence of local C-NOTs within each register, followed by measurement of two communication qubits and subsequent local rotations. Since arbitrary rotations on a single qubit can be performed within a register, the C-NOT operation between different quantum registers is in principle sufficient for universal quantum computation. Similar approaches are also known for deterministic generation of graph states[40] —an essential resource for one-way quantum computation.[41]

3. Robust Operations with Five-Qubit Quantum Registers

In practice, the qubit measurement, initialization, and entanglement generation can be fairly noisy with error probabilities as high as a few percent, due to practical limitations such as finite collection efficiency and poor interferometric stability. As a result, the corresponding error probability in non-local gate circuit will also be very high. In contrast, local unitary operations may fail infrequently ($p_L \lesssim 10^{-4}$) when quantum control techniques for small quantum system are utilized.[2,32–34] We now show that the most important sources of imperfections, such as imperfect initialization, measurement errors for individual qubits in each quantum register, and entanglement generation errors between registers, can be corrected with a modest increase in register size. We determine that with just *three* additional auxiliary qubits and high-fidelity local unitary operations, all these errors can be efficiently suppressed by bit-verification and entanglement purification.[21,22] This provides an extension of Ref. 14 by including imperfections from initialization/measurement, which can be important for physical implemention.[42] In addition, we further improve the entanglement purification scheme,[14] so that it can be more efficient in terms of suppressing both bit and phase errors. Meanwhile, there are other entanglement purification schemes[43,44] that might also be used here.

We will use the following error model for the entire paper: (1) The imperfect local two-qubit operation U_{ij} is

$$U_{ij}\rho U_{ij}^{\dagger} \to (1-p_L)U_{ij}\rho U_{ij}^{\dagger} + \frac{p_L}{4}\mathrm{Tr}_{ij}[\rho]\otimes \mathbf{I}_{ij} \tag{1}$$

where $\mathrm{Tr}_{ij}[\rho]$ is the partial trace over the qubits i and j, and $\mathbf{I}_{ij}$ is the identity operator for qubits i and j. This error model describes that with a probability $1-p_L$ the gates perform the correct operation and with a probability p_L the gates produce a complete random output for the two involved qubits.[a] (2) The imperfect initialization of state $|0\rangle$ will prepare a mixed state

$$\rho_0 = (1-p_I)|0\rangle\langle 0| + p_I|1\rangle\langle 1|, \tag{2}$$

which has error probability p_I, i.e. it prepares the wrong state with a probability p_I. (3) The imperfect measurement of state $|0\rangle$ will correspond to the projection operator

$$P_0 = (1-p_M)|0\rangle\langle 0| + p_M|1\rangle\langle 1|, \tag{3}$$

This operator describes that a qubit prepared in state $|0\rangle$ or $|1\rangle$ will give rise to the opposite measurement output with the measurement error probability p_M. (4) Finally, the entanglement fidelity for a non-ideal preparation is defined as

$$F = \langle\Phi^+|\rho|\Phi^+\rangle, \tag{4}$$

where $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. The infidelity is just $\mathbf{1}-F$. The fidelity does, however, not completely characterize the produced entangled state. Depending on the exact method used to generate the entangled state, one can in some situations argue that the error will predominantly be, e.g. only a phase error,[27,45,46] whereas in other

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[a] The error model introduced in Eq. (1) can be regarded as the worst case error, since it in principle includes all possible errors that can happen to the system.

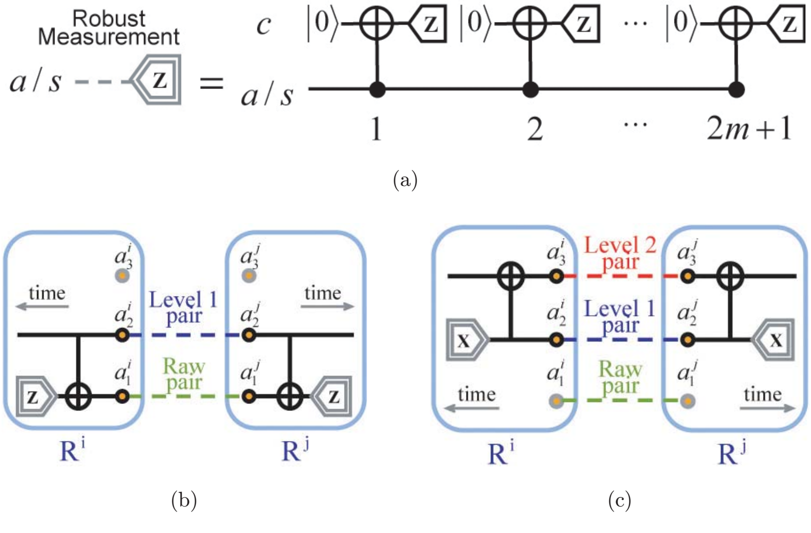
situations it will be a combination of phase and bit flip errors (see Ref. 23 and references therein). Below we shall therefore consider both the situation where we only have a dephasing error as well the situation where we have a more complicated depolarizing error (exact definition given later). As we shall see, the knowledge that the error is of a particular type (e.g. only dephasing error) provides a significant advantage for purification.

We will also assume a separation of error probabilities: any internal, unitary operation within the register fails with extremely low probability, p_L, while all operations connecting the communication qubit to the outside world (initialization, measurement, and entanglement generation) fail with error probabilities that can be several orders of magnitude higher.

$$p_L \ll p_I, p_M, 1 - F. \tag{5}$$

In terms of these quantities, the error probability in the non-local C-NOT gate circuit is of order $p_{CNOT} \sim (1 - F) + 2p_L + 2p_M$. We now show how this fidelity can be greatly increased.

Robust measurement can be implemented by bit-verification: a majority vote among the measurement outcomes (Fig. 2(a)), following a sequence of C-NOT operations between the auxiliary/storage qubit and the communication qubit. (The communication qubit is initialized via optical pumping before each CNOT



(a)

(b)

(c)

Fig. 2. Circuits for robust operations. (a) Robust measurement of the auxiliary/storage qubit, a/s, based on majority vote from $2m + 1$ outcomes of the communication qubit, c. Robust measurement is denoted by the box shown in the upper left corner. (b), (c) Using entanglement pumping to create high fidelity entangled pairs between two registers R^i and R^j. If the two outcomes are the same, it is a successful step of pumping; otherwise generate new pairs and restart the pumping operation from the beginning. The two circuits are for the first level pumping and the second level pumping, purifying bit- and phase-errors, respectively.

operation.) This also allows *robust initialization* by measurement. High-fidelity *robust entanglement generation* is achieved via entanglement purification[21,22,14] (Fig. 2(b) and (c)), in which lower fidelity entanglement between the communication qubits is used to purify entanglement between the auxiliary qubits, which can then be used for the remote C-NOT operation. To make the most efficient use of physical qubits, we introduce a new two-level entanglement pumping scheme. Our circuit (Fig. 2(b)) uses raw Bell pairs to repeatedly purify ("pump") against bit-errors, then the bit-purified Bell pairs are used to pump against phase-errors (Fig. 2(c)).

Entanglement pumping, like entanglement generation, is probabilistic; however, failures are detected. Still, in computation, where each logical gate should be completed within allocated time (clock cycle), failed entanglement pumping can lead to gate failure. Therefore, we should analyze the time required for robust initialization, measurement and entanglement generation, and we will show that the failure probability for these procedures can be made sufficiently small with reasonable time overhead.

3.1. *Robust measurement*

The measurement circuit shown in Fig. 2(a) yields the correct result based on the majority vote from $2m+1$ consecutive readouts. Since the evolution of the system (C-NOT gate) commutes with the measured observable (Z operator) of the auxiliary/storage qubit, it is a quantum non-demolition (QND) measurement, which can be repeated many times. The error probability for the majority vote measurement scheme is:

$$\varepsilon_M \approx \binom{2m+1}{m+1}(p_I + p_M)^{m+1} + \frac{2m+1}{2}p_L. \tag{6}$$

Suppose $p_I = p_M = 5\%$, we can achieve $\varepsilon_M \approx 8 \times 10^{-4}$ by choosing $m^* = 6$ for $p_L = 10^{-4}$, or even $\varepsilon_M \approx 12 \times 10^{-6}$ for $m^* = 10$ and $p_L = 10^{-6}$. Recently, measurement with very high fidelity (ε_M as low as 6×10^{-4}) has been demonstrated in the ion-trap system,[47] using similar ideas as above. The time for robust measurement is

$$\tilde{t}_M = (2m+1)(t_I + t_L + t_M), \tag{7}$$

where t_I, t_L, and t_M are times for initialization, local unitary gate, and measurement, respectively.

3.2. *Robust entanglement generation*

We now use robust measurement and entanglement generation to perform entanglement pumping. For *depolarizing noise*, we apply two-level entanglement pumping. The first level has n_b steps of bit-error pumping using raw Bell pairs (Fig. 2(b)) to produce a bit-error-purified entangled pair. The second level uses these bit-error-purified pairs for n_p steps of phase-error pumping (Fig. 2(c)).

For successful purification, the infidelity of the purified pair, $\varepsilon^{(n_b,n_p)}_{E,\text{infid}}$, depends on both the control parameters (n_b, n_p) and the imperfection parameters (F, p_L, ε_M). For depolarizing error, we find

$$\begin{aligned}\varepsilon^{(n_b\geq 1,n_p\geq 1)}_{E,\text{infid}} &\approx \frac{3+2n_p}{4}p_L + \frac{4+2(n_b+n_p)}{3}(1-F)\varepsilon_M \\ &\quad + (n_p+1)\left(\frac{2(1-F)}{3}\right)^{n_b+1} + \left(\frac{(n_b+1)(1-F)}{3}\right)^{n_p+1} \qquad (8)\end{aligned}$$

to the leading order of p_L and ε_M. The dependence on the initial infidelity $1-F$ is exponentially suppressed at the cost of a linear increase of error from local operations p_L and robust measurement ε_M. Measurement-related errors are suppressed by the prefactor $1-F$, since measurement error does not cause infidelity unless combined with other errors. In the limit of ideal operations ($p_L, \varepsilon_M \to 0$), the infidelity $\varepsilon^{(n_b,n_p)}_{E,\text{infid}}$ can be arbitrarily close to zero.[24] On the other hand, if we use the standard entanglement pumping scheme[21,22] (that alternates purification of bit and phase errors within each pumping level), the reduced infidelity from two-level pumping is always larger than $(1-F)^2/9$. Therefore, for very small p_L and ε_M, the new pumping scheme is crucial to minimize the number of qubits per register.

We remark that a faster and less resource intensive approach may be used if the raw Bell pair is dominated by *dephasing error*. And one-level pumping may be sufficient (i.e. no bit-error purification, $n_b = 0$). For dephasing error, we have

$$\varepsilon^{(0,n_p)}_{E,\text{infid}} \approx (1-F)^{n_p+1} + \frac{2+n_p}{4}p_L + 2(1-F)\varepsilon_M, \qquad (9)$$

by expanding to the leading order of p_L and ε_M.

The overall success probability can be defined as the joint probability that all successive steps succeed. We use the model of finite-state Markov chain[48] to directly calculate the *failure probability* of (n_b, n_p)-two-level entanglement pumping using N_{tot} raw Bell pairs, denoted as $\varepsilon^{(n_b,n_p)}_{E,fail}(N_{\text{tot}})$. See Ref. [24] for detailed analysis.

For given F, p_L, and ε_M, the purified pair has minimum infidelity $\Delta_{\min} = \varepsilon^{(n_b^*,n_p^*)}_{E,\text{infid}}$, obtained by the optimal choice of the control parameters (n_b^*, n_p^*). Then, we calculate the typical value for N_{tot}, by requiring the failure probability and the minimum infidelity to be equal, $\varepsilon^{(n_b^*,n_p^*)}_{E,\text{fail}}(N_{\text{tot}}) = \Delta_{\min}$. The total error probability is

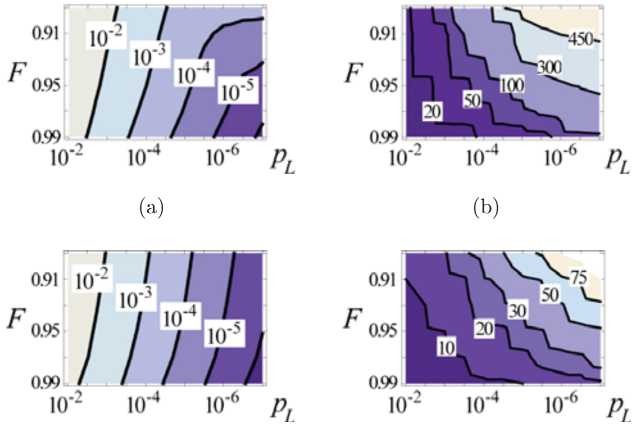
$$\varepsilon_E \approx \varepsilon^{(n_b^*,n_p^*)}_{E,\text{fail}}(N_{\text{tot}}) + \Delta_{\min} = 2\Delta_{\min}. \qquad (10)$$

The total time for robust entanglement generation $\tilde{t}_E$ is

$$\tilde{t}_E \approx \langle N_{\text{tot}}\rangle \times (t_E + t_L + \tilde{t}_M), \qquad (11)$$

where t_E is the average generation time of the raw Bell pair.

Figure 3 shows the contours of ε_E and N_{tot} with respect to the imperfection parameters p_L and $1-F$. We assume $p_I = p_M = 5\%$ for the plot. The choice of p_I and p_M ($< 10\%$) has little effect to the contours, since they only modify ε_M marginally. For initial fidelity $F_0 > 0.95$, the contours of ε_E are almost vertical; that is, ε_E



(a) (b)

(c) (d)

Fig. 3. Contours of the total error probability after purification ε_E (a), (c) and total number of raw Bell pairs consumed N_{tot} (b), (d) with respect to the imperfection parameters p_L (horizontal axis) and F (vertical axis). (a), (b) Two-level pumping is used for depolarizing error, and (c), (d) one-level pumping for dephasing error. $p_I = p_M = 5\%$ is assumed.

is mostly limited by p_L with an overhead factor of about 10. The contours of N_{tot} indicate that the entanglement pumping needs about tens or hundreds of raw Bell pairs to ensure a very high success probability.

4. Architectures Supporting Parallelism

It is important that the architecture of the network-based quantum computer supports parallelism. In particular, it should be able to couple many pairs of qubits that grows linearly with the total number of qubits, as well as simultaneous measurements and local unitary gates. In the following, we analyze an architecture supporting parallelism for the network of NV centers, using (microelectromechanical systems) MEMS devices of mirror arrays and multi-channel detectors, as illustrated in Fig. 4. (A similar architecture has been proposed in Ref. 49.)

The quantum computer operates a diamond sample containing many separately addressable NV centers. Each NV center can be used as a quantum register (left inset) consisting of communication, storage, and auxiliary qubits. The emitted photons from each NV centers can be routed by a set of MEMS-based mirrors,[50] split by the beam splitter, and detected by the two detectors from the multi-channel detectors.

We consider the situation of having as many independently controlled mirrors (and detectors) as the number of NV centers to be manipulated. In that case, it is possible to couple many pairs of NV centers at the same time. For each pair of NV centers, the emitted photons can trigger only two detectors along the routed optical paths and the successful click patterns will generate the entanglement. Since different

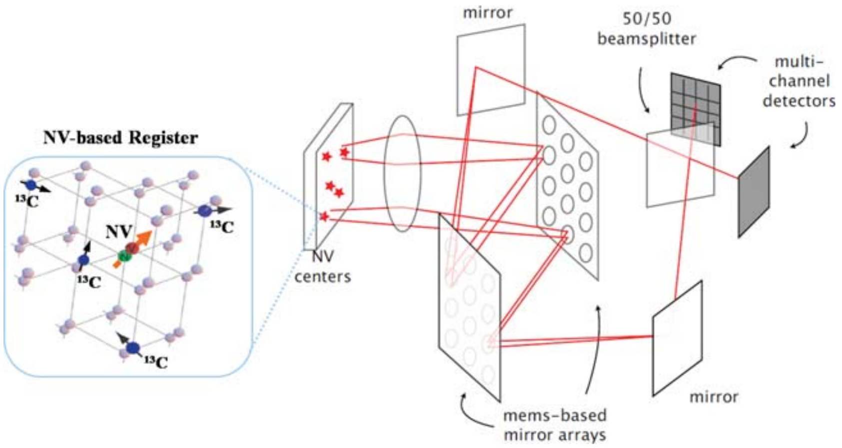


Fig. 4. The architecture of MEMS-based mirror arrays and multi-channel detectors for quantum computer that supports parallelism. The inset illustrates that we can use both the electronic and nuclear spins for the NV-based quantum register.

pairs do not interfere with each other, many pairs of NV centers (qubits) in the computer can be coupled simultaneously. Recently, large scale MEMS-based optical crossconnect switch with more than 1,100 ports has been demonstrated.[50] In addition, MEMS devices with response time less than 0.003 ms has been demonstrated,[51] which is much faster than the total time for robust entanglement generation $\tilde{t}_E$ ($0.1 \sim 1$ ms as estimated below), and we may neglect the operational time associated with the MEMS devices.

We introduce the *clock cycle time*

$$t_C = \tilde{t}_E + 2t_L + \tilde{t}_M \approx \tilde{t}_E, \tag{12}$$

and the *effective error probability*

$$\gamma = \varepsilon_E + 2p_L + 2\varepsilon_M, \tag{13}$$

for general coupling gate between two registers, which can be implemented with a similar approach as the remote C-NOT gate.[39] We now provide an estimate of clock cycle time based on realistic parameters. The time for optical initialization/measurement is $t_I = t_M \approx \ln p_M / \ln(1-\eta)\tau/C$, with photon collection/detection efficiency η, vacuum radiative lifetime τ, and the Purcell factor C for cavity-enhanced radiative decay. We assume that entanglement is generated based on detection of two photons,[26,27] which takes time $t_E \approx (t_I + \tau/C)/\eta^2$. Such two photon schemes can be designed so that the error is primarily phase errors.[26,27,23] If the bit-errors are efficiently suppressed by the intrinsic purification of the entanglement generation scheme, one-level pumping is sufficient; otherwise two-level pumping is needed. Suppose the parameters are $(t_L, \tau, \eta, C) = (0.1\,\mu\text{s}, 10\text{ ns}, 0.2, 10)$[52–54] and $(1-F, p_I, p_M, p_L, \varepsilon_M) = (5\%, 5\%, 5\%, 10^{-6}, 12\times 10^{-6})$. For depolarizing errors, two-level pumping can achieve $(t_C,\gamma) = (997\,\mu\text{s}, 4.5\times 10^{-5})$. If all bit-errors are suppressed by the intrinsic purification of the coincidence scheme, one-level pumping is sufficient and $(t_C,\gamma) = (140\,\mu\text{s}, 3.4\times 10^{-5})$. Finally, t_C should be much shorter than the memory time of the storage qubit, t_{mem}. This is indeed the case for both trapped ions (where $t_{mem}\sim 10$ s has been demonstrated[55,56]) and proximal nuclear spins of NV centers (where t_{mem} approaches 1 s.[11])

This approach yields gates between quantum registers to implement arbitrary quantum circuits. Errors can be further suppressed by using quantum error correction. For example, as shown in Fig. 3, $(p_L, F) = (10^{-4}, 0.95)$ can yield $\gamma \le 2\times 10^{-3}$, well below the 1% threshold for fault tolerant computation for approaches such as the C_4/C_6 code[57] or 2D toric codes[58]; $(p_L, F) = (10^{-6}, 0.95)$ can achieve $\gamma \le 5\times 10^{-5}$, which allows efficient codes such as the BCH [127,43,13] code to be used without concatenation. Following Ref. 59, we estimate 10 registers per logical qubit to be necessary for a calculation involving 10^4 logical qubits and 10^6 logical gates.

5. Conclusion

In summary, we have analyzed a hybrid approach to fault-tolerant quantum computation with optically coupled few-qubit quantum registers. With a reasonable overhead in operational time and gate error probabilities, this approach enables the reduction of an experimental challenge of building a thousand-qubit quantum computer into a more feasible task of optically coupling a thousand five-qubit quantum registers. We have provided an architecture that supports parallel operations for many quantum register pairs at the same time. We further note that it is possible to facilitate fault-tolerant quantum computation with special operations from the hybrid approach such as partial Bell measurement[24] or with systematic optimization using dynamic programming.[60]

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