Filtration and extraction of quantum states from classical inputs

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We propose using a nonlinear Mach-Zehnder interferometer (NMZI) to efficiently prepare photonic quantum states from a classical input. We first analytically investigate the simple NMZI that can filtrate a single-photon state from weak coherent state by preferentially blocking a two-photon component. As a generalization, we show that the cascaded NMZI can deterministically extract an arbitrary quantum state from a strong coherent state. Finally, we numerically demonstrate that the cascaded NMZI can be very efficient in both the input power and the level of cascade. The protocol of quantum state preparation with the NMZI can be extended to various systems of bosonic modes.

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I. INTRODUCTION

Integrated photonics can achieve unprecedented interferometric stability [1,2] and build large-scale interferometers [3,4]. However, reliable quantum state preparation for integrated photonics remains an important challenge because interferometers and coherent input states are insufficient for quantum state preparation. We may use either postselection or nonlinear interaction to overcome this challenge. The approach of postselection only requires linear optical elements and photon detectors, but the preparation of a quantum state is probabilistic and conditioned on the outcome of the projective measurement [5–7]. The approach of nonlinear interaction assisted by an ancillary two-level system (TLS) can deterministically prepare an arbitrary quantum state of the photonic mode [8–10], but it requires strong coupling between the optical mode with a single TLS, which is experimentally challenging for integrated photonics. Alternatively, we may consider using the nonlinear optical waveguide combined with ultrastable interferometers to achieve reliable quantum state preparation without requiring TLS [8–10], postselection [11–13], or feedback or feedforward control [14].

In this paper, we propose to use interferometry combined with Kerr nonlinearity to filtrate single photons or extract any desired quantum states from the coherent state input as illustrated in Fig. 1(a). We first present the idea of quantum state filtration (QSF) of single photons, which keeps the desired single-photon component by *blocking* the undesired component from a different port. We then generalize the idea to quantum state extraction (QSE), which not only keeps the desired component, but also extracts the desired component from the undesired component before blocking or redirecting the residual photons.

II. SINGLE-PHOTON FILTRATION

We first consider the simple task of QSF of single photons. As shown in Fig. 1(b), we use a nonlinear Mach-Zehnder interferometer (NMZI) with a Kerr nonlinear medium in one of the arms. Since Kerr nonlinearity can induce a photon-number-dependent phase shift, we can design the NMZI to induce destructive interference at the output port when there are two photons. More specifically, with a vacuum input at path A (upper path) and a coherent-state input at path B (lower path), the input state to the filtration is

$$|\psi\rangle_{\rm in} = |{\rm vac}\rangle_A \otimes |\alpha\rangle_B, \tag{1}$$

where $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}(b^\dagger)^n|{\rm vac}\rangle$ and $a(a^\dagger)$ and $b(b^\dagger)$ are annihilation (creation) operators for paths A and B, respectively. Each beam splitter (BS) induces a unitary evolution,

$$U_{\rm BS}(\theta_{1,2}) = e^{i\theta_{1,2}(a^\dagger b + ab^\dagger)}, \tag{2}$$

with θ_1 and θ_2 for BS1 and BS2, respectively. The evolution in the nonlinear Kerr medium in path A is

$$U_K(\phi,\varphi) = e^{i\phi a^\dagger a + i\varphi a^\dagger a^\dagger aa}, \tag{3}$$

where φ is the Kerr coefficient and ϕ is linear phase shift (relative to path B). The final output state of the single-photon filtration is

$$\begin{aligned}|\psi\rangle_{\rm out} &= U_{\rm BS}(\theta_2)U_K(\phi,\varphi)U_{\rm BS}(\theta_1)|\psi\rangle_{\rm in}\\ &= \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\mu_{p,q}(a^\dagger)^p(b^\dagger)^q|{\rm vac}\rangle,\end{aligned} \tag{4}$$

with $\mu_{p,q} = \sum_{l=0}^{p}\sum_{n=l}^{l+q}\lambda_{n,p+q-n}\binom{n}{l}\binom{p+q-n}{p-l}(-1)^{n-l}(\sin\theta_2)^{p+n-2l}(\cos\theta_2)^{q-n+2l}$ and $\lambda_{n,m} = \frac{\alpha^{n+m}(\cos\theta_1)^n(-\sin\theta_1)^m}{n!m!}\exp[-|\alpha|^2/2 + in\phi + in(n-1)\varphi]$. The probability of p photons at the output of path A is

$$P_p = \langle p|{\rm Tr}_B\{|\psi_{\rm out}\rangle\langle\psi_{\rm out}|\}|p\rangle = \sum_{q=0}^{\infty} p!q!|\mu_{p,q}|^2. \tag{5}$$

The second-order correlation function [15] is

$$g^{(2)} = \frac{\langle a^\dagger a^\dagger aa\rangle}{\langle a^\dagger a\rangle^2} = \frac{\sum_{p=2}^{\infty}p(p-1)\times P_p}{\left(\sum_{p=1}^{\infty}p\times P_p\right)^2}, \tag{6}$$

which characterizes the generated single-photon state.

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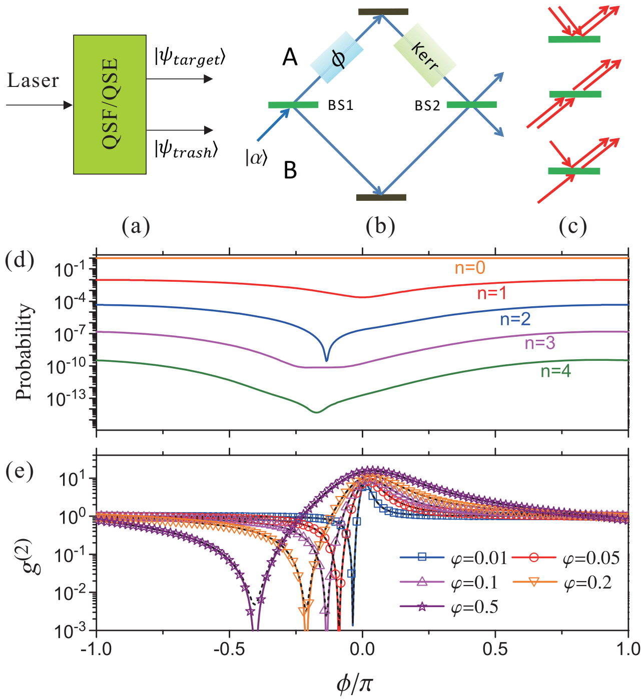

FIG. 1. (a) Schematic of the arbitrary quantum state filtration and extraction from the coherent state input. (b) The configuration for QSF of a single photon from coherent-state input $|\alpha\rangle$ using the simple NMZI (consisting of a Mach-Zehnder interferometer, a Kerr medium, and a phase shifter). (c) Three processes for two-photon output of path A for weak coherent input. (d) The probabilities of n-photons' output of path A against the phase difference between two arms ϕ with $\varphi = 0.1$ and $\alpha = 0.1$. (e) The second-order correlation function ($g^{(2)}$) of light output of path A against ϕ for various φ's with $\alpha = 0.1$. The solid and dashed lines are obtained by the exact numerical and approximated analytical solutions, respectively.

For a weak coherent input $|\alpha|^2 \ll 1$, we have $P_2 \ll P_1$ and can safely neglect the probability of multiple photons ($P_{n\geqslant 3}$). By considering the leading contribution, we have

$$g^{(2)} \approx \frac{2P_2}{P_1^2} \approx \left|\frac{2\mu_{2,0}}{\mu_{1,0}^2}\right|^2 = \left|1 - \frac{1 - e^{i2\varphi}}{(1-\eta e^{-i\phi})^2}\right|^2, \qquad (7)$$

where $\mu_{1,0} = \alpha \cos\theta_1 \cos\theta_2 (e^{i\phi} - \eta)$ and $\mu_{2,0} = \frac{1}{2}(\alpha\cos\theta_1\cos\theta_2)^2[-2\eta e^{i\phi} + e^{i2(\phi+\varphi)} + \eta^2]$ for the simple NMZI with $\eta = \tan\theta_1 \tan\theta_2$. The three terms in $\mu_{2,0}$ correspond to three different processes with two photons at the output of path A as shown in Fig. 1(c). The interference of these three processes can be controlled by the linear phase shift (ϕ) and nonlinear coefficient (φ). The optimal condition for $\mu_{2,0} = 0$ is

$$\eta e^{-i\phi} = 1 \pm \sqrt{1 - e^{i2\varphi}}, \qquad (8)$$

which can always be fulfilled as long as $\varphi \neq 0$ so that the leading contribution to $g^{(2)}$ can be eliminated.

Figure 1(d) shows the probability of n photons at the output of path A (P_n) depending on the linear phase shift ϕ with parameters $\varphi = 0.1$, $\eta = |1 - \sqrt{1 - e^{i2\varphi}}|$, and $\alpha = 0.1$. We find that P_2 is greatly suppressed for $\phi \approx -0.13\pi$, whereas the dominant single-photon emission $P_1 \gg P_{2\text{–}4}$ is not significantly affected. In Fig. 1(e), the relations between $g^{(2)}$ and ϕ are plotted for different values of nonlinear coefficient φ with $\alpha = 0.1$ and η given by the optimal condition from Eq. (8). We find good agreement between the approximated analytical solution from Eqs. (7) and (8) (solid lines) and the exact numerical solution from Eq. (6) (dashed lines). With increasing nonlinear coefficient φ, the deviation from $g^{(2)} = 1$ becomes more significant due to the Fano interference of the three processes [Fig. 1(c)] contributing to $\mu_{2,0}$. These Fano-like curves show a sub-Poissonian statistic with $g^{(2)} \approx 0$ for ϕ close to the optimal condition [Eq. (8)] where the two-photon output can be totally forbidden due to destructive interference. Meanwhile, we can also find the constructive interference of the two-photon output, which gives rise to a super-Poissonian statistic [$g^{(2)}(0) \gg 1$] output. Comparing the curves with different nonlinear effect coefficients, the single-photon filtration is more sensitive to phase ϕ for smaller φ, indicating the crucial role of nonlinearity.

For QSF of the single photon, the fidelity is $F = P_1 = (\cos\theta_1\cos\theta_2)^2|\alpha^2||e^{i2\varphi}|$. The optimal condition requires $\eta = \tan\theta_1\tan\theta_2 \approx 1$, and we have $|\cos\theta_1\cos\theta_2| < \frac{1}{2}$ and $P_1 < \varphi|\alpha^2|/2$, which implies that the fidelity depends on both the Kerr nonlinearity coefficient and the intensity of the coherent-state input. QSF with a simple NMZI cannot suppress the components with $n > 2$ photons [see Fig. 1(d)], and it only works for weak coherent-state $|\alpha^2| \ll 1$, which significantly limits the fidelity. Moreover, the fidelity of QSF is fundamentally limited by the overlap between the input state and the target state $P_{\rm succ} < |\langle\psi_{\rm out}|\psi_{\rm in}\rangle|^2$ because it blocks all undesired components. To go beyond this limit, we need to generalize QSF to QSE, which not only keeps the desired component, but also extracts the desired component from the undesired ones.

III. QUANTUM STATE EXTRACTION

To implement QSE, we consider the cascaded NMZI with a series of NMZIs connected sequentially. As shown in Fig. 2(a), the basic element consists of a BS (θ) followed by a linear phase shifter (ϕ) and a Kerr medium (φ) in the upper path. The basic element can be represented by a standard two-port unitary [Fig. 2(b)],

$$U(\phi,\varphi,\theta) = U_{\rm K}(\phi,\varphi)U_{\rm BS}. \qquad (9)$$

The cascaded MNZI with N elements can be characterized by

$$U_N = \prod_{l=1}^{N} U(\phi_l,\varphi_l,\theta_l). \qquad (10)$$

For example, the simple NMZI [Fig. 1(b)] consists of $N = 2$ basic elements with $\phi_2 = \varphi_2 = 0$.

The cascaded NMZI cannot only keep the desired single-photon component, but also extract the (desired) single-photon state from (undesired) multiphoton states as long as there are enough photons in the undesired component. We numerically optimize the fidelity by tuning the parameters of the N elements. As illustrated in Fig. 2(c), the optimized fidelity of single-photon extraction $F = P_1$ increases with N

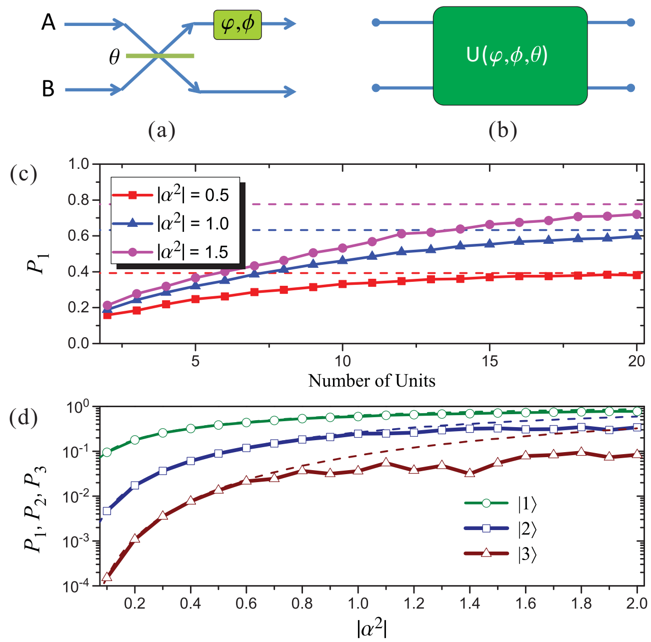

FIG. 2. Cascaded NMZI. (a) The basic element consists of a BS (θ) followed by a linear phase shifter (ϕ) and a Kerr medium (φ) in the upper path. (b) Schematic of the element for cascaded NMZI. (c) Fidelity of the single-photon extraction increases with the number of cascade elements N. The parameters are optimized numerically under the constraint $P_{n\geqslant 2} < 0.01$ with $\varphi = 0.1$. (d) Fidelity of a Fock state extraction ($n = 1$–3) increase with $|\alpha|^2$ for cascaded NMZI with $N = 40$. The results are obtained by optimizing the parameters of each unit under the constraint that $1 - P_0 - P_n \leqslant 0.01$.

monotonically with asymptotic value $F \rightarrow 1 - |\langle 0|\alpha\rangle|^2$ (dashed lines) because our passive device cannot extract a single photon from the vacuum component. Furthermore, the cascaded NMZI can extract Fock state $|n\rangle$ with $n = $–$3, \ldots$. The asymptotic fidelity of n-photon extraction is $F \rightarrow 1 - \sum_{m=0}^{n-1} |\langle m|\alpha\rangle|^2$, which can be achieved for $|\alpha|^2 \leqslant 1.5/n$ with a cascaded NMZI of $N = 40$ elements, as shown in Fig. 2(d).

Remarkably, the cascaded NMZI can extract *arbitrary* superposition of Fock states with a large coherent-state input ($|\alpha| \gg 1$) with almost perfect fidelity. For $\theta_l \ll 1$ with $l = 1, \ldots, N$, almost all input photons will be guided in path B, which effectively remains as a coherent state [with a small deviation of $O(\theta)$] for all intermediate stages. The effect of each beam splitter on the upper path can be regarded as an effective displacement operation to path A as $D(\epsilon_l) = e^{\epsilon_l a^\dagger - \epsilon_l^* a}$ with $\epsilon_l = \alpha\theta_l$ and a small deviation of $O(\epsilon_l^2/\alpha^2)$ [16]. In addition, the linear phase shift and Kerr nonlinearity can achieve the unitary evolution $U_K(\phi_l,\varphi) = e^{i\phi_l a^\dagger a + i\varphi a^\dagger a^\dagger a a}$. Hence, the cascaded NMZI of N elements can induce the unitary evolution

$$U_K(\phi_N,\varphi)U(\epsilon_N)\cdots U_K(\phi_2,\varphi)U(\epsilon_2)U_K(\phi_1,\varphi)U(\epsilon_1),$$

which in principle can accomplish any desired unitary transformation for sufficiently large N and carefully chosen $\{\phi_l,\epsilon_l\}_{l=1,\ldots,N}$ [17–19]. Despite the large overhead in N, this provides a generic approach using the cascaded NMZI to extract the arbitrary superposition of Fock states from a large coherent state with almost perfect fidelity.

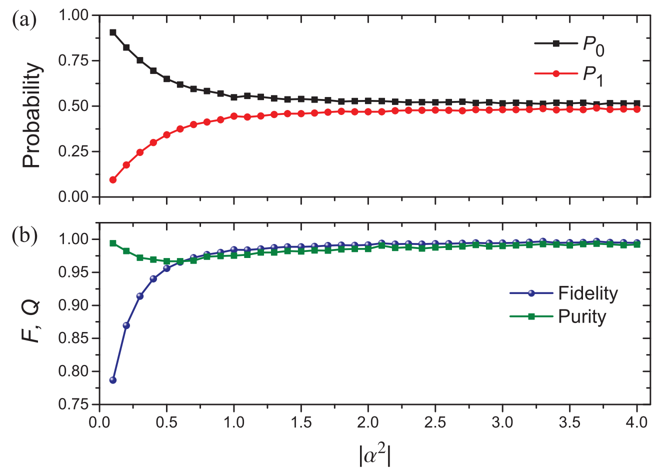

FIG. 3. QSE for $|\psi_{\text{target}}\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. (a) The probability of $|0\rangle$ and $|1\rangle$, (b) the fidelity (F) and purity (Q) of the output. The results are obtained by optimizing the parameters of a chain of $N = 20$ units under the constraint that $1 - P_0 - P_1 \leqslant 0.01$.

In practice, it is favorable to design the cascaded NMZI with a small number of elements. To illustrate the feasibility, we consider the target state $|\psi_{\text{target}}\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ using $N = 20$ cascaded elements optimize the fidelity by tuning parameters of $\{\varphi_l,\phi_l,\theta_l\}_{l=1,\ldots,N}$. As illustrated in Fig. 3, we can improve the fidelity F and purity $Q = \mathrm{Tr}(\rho_A^2)$ of the extracted state by increasing $|\alpha|^2$. Both F and Q are greater than 97.5% when $|\alpha|^2 \geqslant 1.5$. It is intriguing that a high fidelity QSE of a superposition of Fock states can be achieved using a reasonable size coherent state and a finite-stage cascaded NMZI.

IV. DISCUSSION

The photonic integrated circuits provide a promising platform for realizing the QSF or QSE where arrays of beam splitters and phase shifters can be integrated on a chip [3,4]. In the experiments, the most challenging part is the Kerr nonlinear at the single-photon level ($\varphi = 0.1$ in this paper). One feasible approach for realizing such strong Kerr nonlinearity is taking advantage of the collective effect of the atomic ensemble. Recently, the single-photon level nonlinearity has been demonstrated by interfacing the atomic ensemble and photonic waveguide [20–22], such as a nanofiber [23,24], hollow-core photonic crystal fiber [25,26], and integrated waveguide [27]. Therefore, the QSF or QSE can be realized in the photonic integrated chip by trapping the atom clouds close to the chip. Alternatively, the strong Kerr nonlinearity can be achieved by incorporating the materials with high intrinsic nonlinearity into the photonic chip [28,29], and the single-photon level nonlinear effect might be realized by new materials, such as graphene [30] and a topological insulator [31].

The idea of QSE can be extended from optical frequency to microwave and terahertz frequencies. In particular, the superconducting quantum circuits [32,33] can readily realize the QSE by using the strong nonlinearity of superconducting qubits. In addition, the mechanism of the QSE is very general and can also be generalized to other collective bosonic excitations in solids, such as a surface plasmon [34], exciton-polariton [35], magnon [36,37], and phonon [38]. For example, the phononic quantum states can be engineered by coupling the high quality mechanical oscillators with superconducting qubits [38] or using the intrinsic mechanical nonlinearity of mechanical resonators [39].

V. CONCLUSION

We have demonstrated that the simple NMZI can filtrate a single-photon state from a weak coherent state. Using cascaded NMZI, we can reliably extract an arbitrary quantum state from a strong coherent state. Since our scheme only requires Kerr nonlinearity, a linear phase shifter, and a beam splitter, it can be implemented in superconducting circuits, coupled optomechanical systems, as well as photonic integrated circuits.

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